



Practice Problems

Problems based on centre of mass

► Basic level

- Where will be the centre of mass on combining two masses m and M ($M > m$) [RPET 2003]
(a) Towards m (b) Towards M (c) Between m and M (d) Anywhere
- Two objects of masses 200 gm and 500 gm possess velocities $10\hat{i}\text{ m/s}$ and $3\hat{i} + 5\hat{j}\text{ m/s}$ respectively. The velocity of their centre of mass in m/s is
(a) $5\hat{i} - 25\hat{j}$ (b) $\frac{5}{7}\hat{i} - 25\hat{j}$ (c) $5\hat{i} + \frac{25}{7}\hat{j}$ (d) $25\hat{i} - \frac{5}{7}\hat{j}$
- In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 \AA ($1\text{ \AA} = 10^{-10}\text{ m}$). The approximate location of the centre of mass of the molecule from hydrogen is (assuming the chlorine atom to be about 35.5 times massive as hydrogen) [Kerala (Engg.) 2002]
(a) 1 \AA (b) 2.5 \AA (c) 1.24 \AA (d) 1.5 \AA
- Four particles of masses m , $2m$, $3m$ and $4m$ are arranged at the corners of a parallelogram with each side equal to a and one of the angles between two adjacent sides is 60° . The parallelogram lies in the x - y plane with mass m at the origin and $4m$ on the x -axis. The centre of mass of the arrangement will be located at
(a) $\left(\frac{\sqrt{3}}{2}a, 0.95a\right)$ (b) $\left(0.95a, \frac{\sqrt{3}}{4}a\right)$ (c) $\left(\frac{3a}{4}, \frac{a}{2}\right)$ (d) $\left(\frac{a}{2}, \frac{3a}{4}\right)$
- A system consists of 3 particles each of mass m and located at $(1, 1)$, $(2, 2)$, $(3, 3)$. The co-ordinates of the centre of mass are
(a) $(6, 6)$ (b) $(3, 3)$ (c) $(2, 2)$ (d) $(1, 1)$
- If a bomb is thrown at a certain angle with the horizontal and after exploding on the way the different fragments move in different directions then the centre of mass
(a) Would move along the same parabolic path (b) Would move along a horizontal path
(c) Would move along a vertical line (d) None of these
- Four identical spheres each of mass m are placed at the corners of a square of side 2 metre . Taking the point of intersection of the diagonals as the origin, the co-ordinates of the centre of mass are
(a) $(0, 0)$ (b) $(1, 1)$ (c) $(-1, 1)$ (d) $(1, -1)$

►► Advance level

- Two particles A and B initially at rest move towards each other under a mutual force of attraction. At the instant when the speed of A is v and the speed of B is $2v$, the speed of centre of mass of the system is
(a) Zero (b) v (c) $1.5v$ (d) $3v$
- A circular plate of uniform thickness has diameter 56 cm . A circular part of diameter 42 cm is removed from one edge. What is the position of the centre of mass of the remaining part
(a) 3 cm (b) 6 cm (c) 9 cm (d) 12 cm



10. Two point masses m and M are separated by a distance L . The distance of the centre of mass of the system from m is
- (a) $L(m/M)$ (b) $L(M/m)$ (c) $L\left(\frac{M}{m+M}\right)$ (d) $L\left(\frac{m}{m+M}\right)$
11. Three identical spheres, each of mass 1 kg are placed touching each other with their centres on a straight line. Their centres are marked K , L and M respectively. The distance of centre of mass of the system from K is
- (a) $\frac{KL + KM + LM}{3}$ (b) $\frac{KL + KM}{3}$ (c) $\frac{KL + LM}{3}$ (d) $\frac{KM + LM}{3}$
12. Two particles of masses 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2 m/s , their centre of mass has a velocity of 0.5 m/s . When the relative velocity of approach becomes 3 m/s , the velocity of the centre of mass is
- (a) 0.5 m/s (b) 0.75 m/s (c) 1.25 m/s (d) Zero

Problems based on angular displacement, velocity and acceleration

► Basic level

13. In rotational motion of a rigid body, all particle move with
- (a) Same linear and angular velocity (b) Same linear and different angular velocity
(c) With different linear velocities and same angular velocities (d) With different linear velocities and different angular velocities
14. The angular speed of a fly-wheel making 120 revolution/minute is [Pb. PMT 1999; CPMT 2002]
- (a) $\pi\text{ rad/sec}$ (b) $2\pi\text{ rad/sec}$ (c) $4\pi\text{ rad/sec}$ (d) $4\pi^2\text{ rad/sec}$
15. A flywheel gains a speed of 540 r.p.m. in 6 sec. Its angular acceleration will be [KCET (Med.) 2001]
- (a) $3\pi\text{ rad/sec}^2$ (b) $9\pi\text{ rad/sec}^2$ (c) $18\pi\text{ rad/sec}^2$ (d) $54\pi\text{ rad/sec}^2$
16. A car is moving at a speed of 72 km/hr. the diameter of its wheels is 0.5 m. If the wheels are stopped in 20 rotations by applying brakes, then angular retardation produced by the brakes is
- (a) -25.5 rad/s^2 (b) -29.5 rad/s^2 (c) -33.5 rad/s^2 (d) -45.5 rad/s^2
17. A wheel is rotating at 900 r.p.m. about its axis. When the power is cut-off, it comes to rest in 1 minute. The angular retardation in radian/s^2 is
- (a) $\pi/2$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/8$

►► Advance level

18. A particle B is moving in a circle of radius a with a uniform speed u . C is the centre of the circle and AB is diameter. The angular velocity of B about A and C are in the ratio
- (a) $1 : 1$ (b) $1 : 2$ (c) $2 : 1$ (d) $4 : 1$
19. Two particles having mass ' M ' and ' m ' are moving in circular paths having radii R and r . If their time periods are same then the ratio of their angular velocities will be
- (a) $\frac{r}{R}$ (b) $\frac{R}{r}$ (c) 1 (d) $\sqrt{\frac{R}{r}}$
20. A body is in pure rotation. The linear speed v of a particle, the distance r of the particle from the axis and angular velocity ω of the body are related as $\omega = \frac{v}{r}$, thus
- (a) $\omega \propto \frac{1}{r}$ (b) $\omega \propto r$ (c) $\omega = 0$ (d) ω is independent of r



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21. A strap is passing over a wheel of radius 30 cm. During the time the wheel moving with initial constant velocity of 2 rev/sec. comes to rest the strap covers a distance of 25 m. The deceleration of the wheel in rad/s^2 is
 (a) 0.94 (b) 1.2 (c) 2.0 (d) 2.5
22. A particle starts rotating from rest. Its angular displacement is expressed by the following equation $\theta = 0.025 t^2 - 0.1t$ where θ is in radian and t is in seconds. The angular acceleration of the particle is
 (a) 0.5 rad/sec^2 at the end of 10 sec (b) 0.3 rad/sec^2 at the end of 2 sec
 (c) 0.05 rad/sec^2 at the end of 1 sec (d) Constant 0.05 rad/sec^2
23. The planes of two rigid discs are perpendicular to each other. They are rotating about their axes. If their angular velocities are 3 rad/sec and 4 rad/sec respectively, then the resultant angular velocity of the system would be
 (a) 1 rad/sec (b) 7 rad/sec (c) 5 rad/sec (d) $\sqrt{12} \text{ rad/sec}$
24. A sphere is rotating about a diameter
 (a) The particles on the surface of the sphere do not have any linear acceleration
 (b) The particles on the diameter mentioned above do not have any linear acceleration
 (c) Different particles on the surface have different angular speeds
 (d) All the particles on the surface have same linear speed
25. A rigid body is rotating with variable angular velocity $(a - bt)$ at any instant of time t . The total angle subtended by it before coming to rest will be (a and b are constants)
 (a) $\frac{(a-b)a}{2}$ (b) $\frac{a^2}{2b}$ (c) $\frac{a^2 - b^2}{2b}$ (d) $\frac{a^2 - b^2}{2a}$
26. When a ceiling fan is switched on, it makes 10 rotations in the first 3 seconds. How many rotations will it make in the next 3 seconds (Assume uniform angular acceleration)
 (a) 10 (b) 20 (c) 30 (d) 40
27. When a ceiling fan is switched off, its angular velocity falls to half while it makes 36 rotations. How many more rotations will it make before coming to rest (Assume uniform angular retardation)
 (a) 36 (b) 24 (c) 18 (d) 12
28. Let \vec{A} be a unit vector along the axis of rotation of a purely rotating body and \vec{B} be a unit vector along the velocity of a particle P of the body away from the axis. The value of $\vec{A} \cdot \vec{B}$ is
 (a) 1 (b) -1 (c) 0 (d) None of these

Problems based on torque, couple

► Basic level

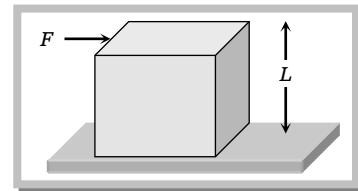
29. Let F be the force acting on a particle having position vector \vec{r} and \vec{T} be the torque of this force about the origin. Then [AIEEE 2003]
 (a) $\vec{r} \cdot \vec{T} = 0$ and $\vec{F} \cdot \vec{T} = 0$ (b) $\vec{r} \cdot \vec{T} = 0$ and $\vec{F} \cdot \vec{T} \neq 0$
 (c) $\vec{r} \cdot \vec{T} \neq 0$ and $\vec{F} \cdot \vec{T} = 0$ (d) $\vec{r} \cdot \vec{T} \neq 0$ and $\vec{F} \cdot \vec{T} \neq 0$
30. A couple produces [CBSE PMT 1997]
 (a) Purely linear motion (b) Purely rotational motion
 (c) Linear and rotational motion (d) No motion
31. For a system to be in equilibrium, the torques acting on it must balance. This is true only if the torques are taken about
 (a) The centre of the system (b) The centre of mass of the system



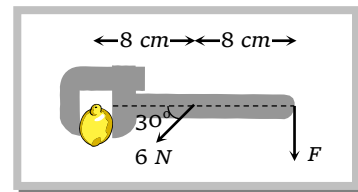
- (c) Any point on the system outside it (d) Any point on the system or outside it
32. What is the torque of the force $\vec{F} = (2\hat{i} - 3\hat{j} + 4\hat{k})N$ acting at the pt. $\vec{r} = (3\hat{i} + 2\hat{j} + 3\hat{k})m$ about the origin
 (a) $-17\hat{i} + 6\hat{j} + 13\hat{k}$ (b) $-6\hat{i} + 6\hat{j} - 12\hat{k}$ (c) $17\hat{i} - 6\hat{j} - 13\hat{k}$ (d) $6\hat{i} - 6\hat{j} + 12\hat{k}$
33. Two men are carrying a uniform bar of length L , on their shoulders. The bar is held horizontally such that younger man gets $(1/4)th$ load. Suppose the younger man is at the end of the bar, what is the distance of the other man from the end
 (a) $L/3$ (b) $L/2$ (c) $2L/3$ (d) $3L/4$
34. A uniform meter scale balances at the 40 cm mark when weights of 10 g and 20 g are suspended from the 10 cm and 20 cm marks. The weight of the metre scale is
 (a) 50 g (b) 60 g (c) 70 g (d) 80 g

►► **Advance level**

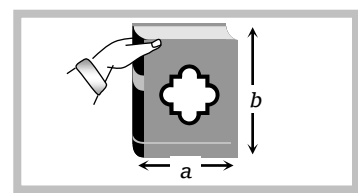
35. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ . A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



- (a) Infinitesimal
 (b) $mg/4$
 (c) $mg/2$
 (d) $mg(1 - \mu)$
36. When a force of 6.0 N is exerted at 30° to a wrench at a distance of 8 cm from the nut, it is just able to loosen the nut. What force F would be sufficient to loosen it, if it acts perpendicularly to the wrench at 16 cm from the nut



- (a) 3 N
 (b) 6 N
 (c) 4 N
 (d) 1.5 N
37. A person supports a book between his finger and thumb as shown (the point of grip is assumed to be at the corner of the book). If the book has a weight of W then the person is producing a torque on the book of



- (a) $W \frac{a}{2}$ anticlockwise
 (b) $W \frac{b}{2}$ anticlockwise
 (c) Wa anticlockwise
 (d) Wa clockwise
38. Weights of $1\text{ g}, 2\text{ g}, \dots, 100\text{ g}$ are suspended from the $1\text{ cm}, 2\text{ cm}, \dots, 100\text{ cm}$, marks respectively of a light metre scale. Where should it be supported for the system to be in equilibrium
 (a) 55 cm mark (b) 60 cm mark (c) 66 cm mark (d) 72 cm mark

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39. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. The minimum value of F for which the cube begins to tilt about the edge is (assume that the cube does not slide)
- (a) $\frac{mg}{4}$ (b) $\frac{2mg}{3}$ (c) $\frac{3mg}{4}$ (d) mg

Problems based on moment of inertia

► Basic level

40. A circular disc of radius R and thickness $\frac{R}{6}$ has moment of inertia I about an axis passing through its centre and perpendicular to its plane. It is melted and recasted into a solid sphere. The moment of inertia of the sphere about its diameter as axis of rotation is

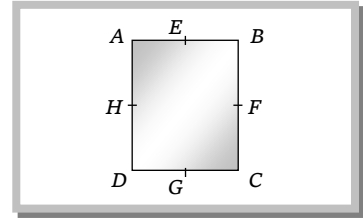
[EAMCET 2003]

- (a) I (b) $\frac{2I}{8}$ (c) $\frac{I}{5}$ (d) $\frac{I}{10}$
41. The moment of inertia of a meter scale of mass 0.6 kg about an axis perpendicular to the scale and located at the 20 cm position on the scale in kg m^2 is (Breadth of the scale is negligible)
- (a) 0.074 (b) 0.104 (c) 0.148 (d) 0.208
42. Two discs of the same material and thickness have radii 0.2 m and 0.6 m . Their moments of inertia about their axes will be in the ratio
- [MP PET 2003]
- (a) $1 : 81$ (b) $1 : 27$ (c) $1 : 9$ (d) $1 : 3$
43. A circular disc is to be made by using iron and aluminium, so that it acquires maximum moment of inertia about its geometrical axis. It is possible with
- (a) Iron and aluminium layers in alternate order (b) Aluminium at interior and iron surrounding it
(c) Iron at interior and aluminium surrounding it (d) Either (a) or (c)
44. The moment of inertia of semicircular ring about its centre is
- (a) MR^2 (b) $\frac{MR^2}{2}$ (c) $\frac{MR^2}{4}$ (d) None of these
45. Moment of inertia of a disc about its own axis is I . Its moment of inertia about a tangential axis in its plane is [MP PMT 2002]
- (a) $\frac{5}{2}I$ (b) $3I$ (c) $\frac{3}{2}I$ (d) $2I$
46. A wheel of mass 10 kg has a moment of inertia of 160 kg-m^2 about its own axis, the radius of gyration will be [Pb. PMT 2002]
- (a) 10 m (b) 8 m (c) 6 m (d) 4 m
47. Four particles each of mass m are placed at the corners of a square of side length l . The radius of gyration of the system about an axis perpendicular to the square and passing through its centre is
- (a) $\frac{l}{\sqrt{2}}$ (b) $\frac{l}{2}$ (c) l (d) $(\sqrt{2})l$
48. The moment of inertia of a rod (length l , mass m) about an axis perpendicular to the length of the rod and passing through a point equidistant from its mid point and one end is
- (a) $\frac{ml^2}{12}$ (b) $\frac{7}{48}ml^2$ (c) $\frac{13}{48}ml^2$ (d) $\frac{19}{48}ml^2$
49. Three point masses m_1, m_2, m_3 are located at the vertices of an equilateral triangle of length ' a '. The moment of inertia of the system about an axis along the altitude of the triangle passing through m_1 is
- (a) $(m_2 + m_3)\frac{a^2}{4}$ (b) $(m_1 + m_2 + m_3)a^2$ (c) $(m_1 + m_2)\frac{a^2}{2}$ (d) $(m_2 + m_3)a^2$



50. In a rectangle $ABCD$ ($BC = 2AB$). The moment of inertia along which axis will be minimum

- (a) BC
- (b) BD
- (c) HF
- (d) EG

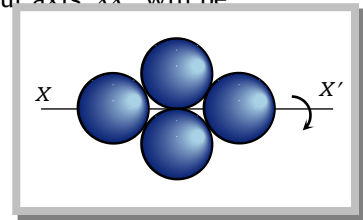


51. Two loops P and Q are made from a uniform wire. The radii of P and Q are r_1 and r_2 respectively, and their moments of inertia are I_1 and I_2 respectively. If $I_2/I_1 = 4$ then $\frac{r_2}{r_1}$ equals

- (a) $4^{2/3}$
- (b) $4^{1/3}$
- (c) $4^{-2/3}$
- (d) $4^{-1/3}$

52. The moment of inertia of a sphere (mass M and radius R) about its diameter is I . Four such spheres are arranged as shown in the figure. The moment of inertia of the system about axis XX' will be

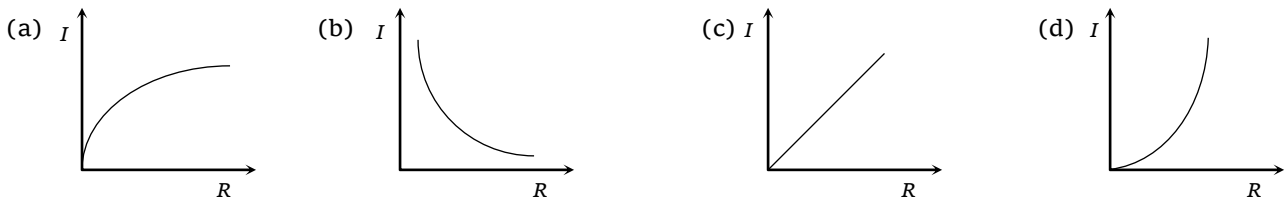
- (a) $3I$
- (b) $5I$
- (c) $7I$
- (d) $9I$



53. Three identical thin rods each of length l and mass M are joined together to form a letter H . What is the moment of inertia of the system about one of the sides of H

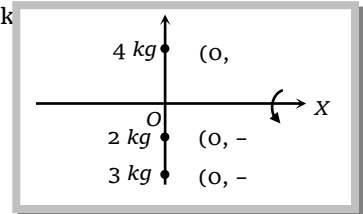
- (a) $\frac{Ml^2}{3}$
- (b) $\frac{Ml^2}{4}$
- (c) $\frac{2Ml^2}{3}$
- (d) $\frac{4Ml^2}{3}$

54. Moment of inertia of a sphere of mass M and radius R is I . Keeping M constant if a graph is plotted between I and R , then its form would be



55. Three particles are situated on a light and rigid rod along Y axis as shown in the figure. If the system is rotating with an angular velocity of 2 rad/sec about X axis, then the total kinetic energy is

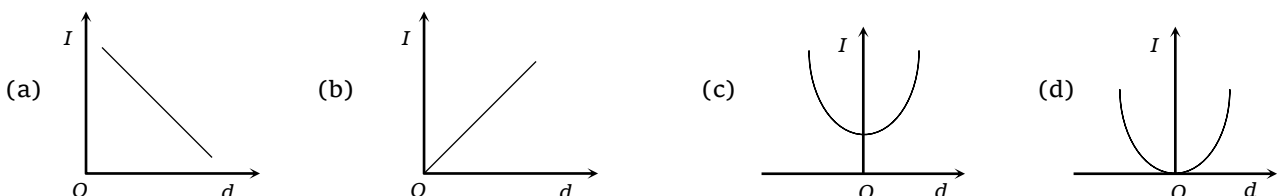
- (a) $92 J$
- (b) $184 J$
- (c) $276 J$
- (d) $46 J$



56. On account of melting of ice at the north pole the moment of inertia of spinning earth

- (a) Increases
- (b) Decreases
- (c) Remains unchanged
- (d) Depends on the time

57. According to the theorem of parallel axes $I = I_g + Md^2$, the graph between I and d will be

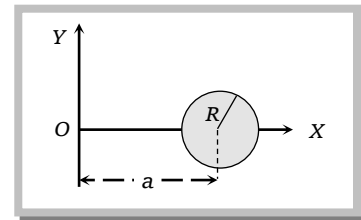


58. What is the moment of inertia of a square sheet of side l and mass per unit area μ about an axis passing through the centre and perpendicular to its plane

- (a) $\frac{\mu l^2}{12}$ (b) $\frac{\mu l^2}{6}$ (c) $\frac{\mu l^4}{12}$ (d) $\frac{\mu l^4}{6}$

59. The adjoining figure shows a disc of mass M and radius R lying in the X - Y plane with its centre on X -axis at a distance a from the origin. Then the moment of inertia of the disc about the X -axis is

- (a) $M\left(\frac{R^2}{2}\right)$
 (b) $M\left(\frac{R^2}{4}\right)$
 (c) $M\left(\frac{R^2}{4} + a^2\right)$
 (d) $M\left(\frac{R^2}{2} + a^2\right)$



60. We have two spheres, one of which is hollow and the other solid. They have identical masses and moment of inertia about their respective diameters. The ratio of their radius is given by

- (a) 5 : 7 (b) 3 : 5 (c) $\sqrt{3} : \sqrt{5}$ (d) $\sqrt{3} : \sqrt{7}$

►► **Advance level**

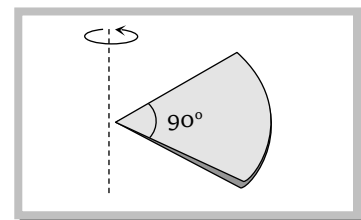
61. From a uniform wire, two circular loops are made (i) P of radius r and (ii) Q of radius nr . If the moment of inertia of Q about an axis passing through its centre and perpendicular to its plane is 8 times that of P about a similar axis, the value of n is (diameter of the wire is very much smaller than r or nr)

- (a) 8 (b) 6 (c) 4 (d) 2

62. One quarter sector is cut from a uniform circular disc of radius R . This sector has mass M . It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is

[IIT-JEE (Screening) 2001]

- (a) $\frac{1}{2}MR^2$
 (b) $\frac{1}{4}MR^2$
 (c) $\frac{1}{8}MR^2$
 (d) $\sqrt{2}MR^2$



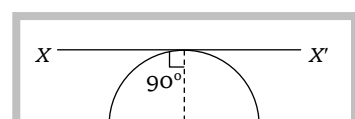
63. Two discs of same thickness but of different radii are made of two different materials such that their masses are same. The densities of the materials are in the ratio 1 : 3. The moments of inertia of these discs about the respective axes passing through their centres and perpendicular to their planes will be in the ratio

- (a) 1 : 3 (b) 3 : 1 (c) 1 : 9 (d) 9 : 1

64. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is

[IIT-JEE (Screening) 2000]

- (a) $\frac{\rho L^3}{8\pi^2}$



(b) $\frac{\rho L^3}{16\pi^2}$

(c) $\frac{5\rho L^3}{16\pi^2}$

(d) $\frac{3\rho L^3}{8\pi^2}$

65. If solid sphere and solid cylinder of same radius and density rotate about their own axis, the moment of inertia will be greater for ($L = R$)

- (a) Solid sphere (b) Solid cylinder (c) Both (d) Equal both

66. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum is located at a distance of **[IIT-JEE 1995; AIIMS 2000]**

- (a) 0.4 m from mass of 0.3 kg (b) 0.98 from mass of 0.3 kg
 (c) 0.70 m from mass of 0.7 kg (d) 0.98 m from mass of 0.7 kg

kg

67. A circular disc A of radius r is made from an iron plate of thickness t and another circular disc B of radius $4r$ is made from an iron plate of thickness $t/4$. The relation between the moments of inertia I_A and I_B is

- (a) $I_A > I_B$ (b) $I_A = I_B$
 (c) $I_A < I_B$ (d) Depends on the actual values of t and r

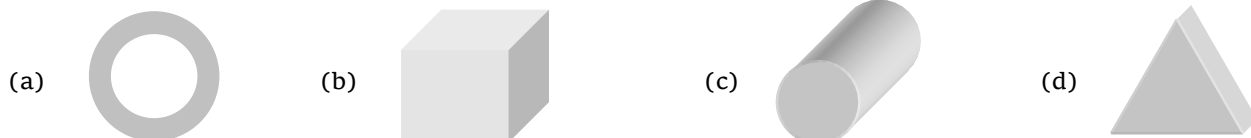
68. A thin wire of length l and mass M is bent in the form of a semi-circle. What is its moment of inertia about an axis passing through the ends of the wire

- (a) $\frac{Ml^2}{2}$ (b) $\frac{Ml^2}{\pi^2}$ (c) $\frac{2Ml^2}{\pi^2}$ (d) $\frac{Ml^2}{2\pi^2}$

69. If I_1 is the moment of inertia of a thin rod about an axis perpendicular to its length and passing through its centre of mass, and I_2 is the moment of inertia of the ring formed by bending the rod, then

- (a) $I_1 : I_2 = 1 : 1$ (b) $I_1 : I_2 = \pi^2 : 3$ (c) $I_1 : I_2 = \pi : 4$ (d) $I_1 : I_2 = 3 : 5$

70. Four solids are shown in cross section. The sections have equal heights and equal maximum widths. They have the same mass. The one which has the largest rotational inertia about a perpendicular through the centre of mass is

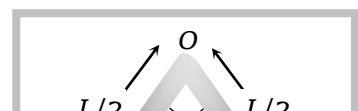


71. The moment of inertia I of a solid sphere having fixed volume depends upon its volume V as

- (a) $I \propto V$ (b) $I \propto V^{2/3}$ (c) $I \propto V^{5/3}$ (d) $I \propto V^{3/2}$

72. A thin rod of length L and mass M is bent at the middle point O at an angle of 60° as shown in figure. The moment of inertia of the rod about an axis passing through O and perpendicular to the plane of the rod will be

(a) $\frac{ML^2}{6}$



- (b) $\frac{ML^2}{12}$
 (c) $\frac{ML^2}{24}$
 (d) $\frac{ML^2}{3}$

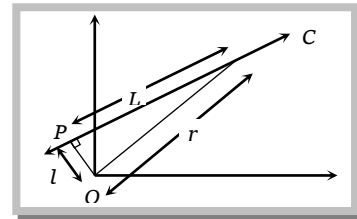
Problems based on angular momentum

► Basic level

73. The motion of planets in the solar system is an example of the conservation of [AIIMS 2003]
 (a) Mass (b) Linear momentum (c) Angular momentum (d) Energy

74. A disc is rotating with an angular speed of ω . If a child sits on it, which of the following is conserved
 (a) Kinetic energy (b) Potential energy (c) Linear momentum (d) Angular momentum

75. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about O



[AIEEE 2002]

- (a) mvL
 (b) $mv l$
 (c) mvr
 (d) Zero

76. Two rigid bodies A and B rotate with rotational kinetic energies E_A and E_B respectively. The moments of inertia of A and B about the axis of rotation are I_A and I_B respectively. If $I_A = I_B/4$ and $E_A = 100 E_B$ the ratio of angular momentum (L_A) of A to the angular momentum (L_B) of B is

- (a) 25 (b) 5/4 (c) 5 (d) 1/4

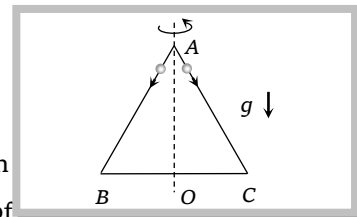
77. A uniform heavy disc is rotating at constant angular velocity ω about a vertical axis through its centre and perpendicular to the plane of the disc. Let L be its angular momentum. A lump of plasticine is dropped vertically on the disc and sticks to it. Which will be constant

[AMU (Med.) 2001]

- (a) ω (b) ω and L both (c) L only (d) Neither ω nor L

78. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A . The triangle is set rotating about the vertical axis AO . Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are

[IIT-JEE (Screening) 2000]



- (a) Angular velocity and total energy (kinetic and potential)
 (b) Total angular momentum and total energy
 (c) Angular velocity and moment of inertia about the axis of rotation
 (d) Total angular momentum and moment of inertia about the axis of

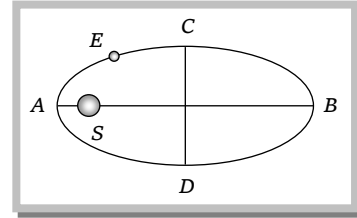
79. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω . Two objects each of mass m are attached gently to the opposite ends of a diameter of the ring. The ring will now rotate with an angular velocity



- (a) $\frac{\omega(M-2m)}{M+2m}$ (b) $\frac{\omega M}{M+2m}$ (c) $\frac{\omega M}{M+m}$ (d) $\frac{\omega(M+2m)}{M}$

80. The earth E moves in an elliptical orbit with the sun S at one of the foci as shown in the figure. Its speed of motion will be maximum at the point

- (a) C
 (b) A
 (c) B
 (d) D



81. A rigid spherical body is spinning around an axis without any external torque. Due to change in temperature, the volume increases by 1%. Its angular speed

- (a) Will increase approximately by 1% (b) Will decrease approximately by 1%
 (c) Will decrease approximately by 0.67% (d) Will decrease approximately by 0.33%

82. A uniform disc of mass M and radius R is rotating about a horizontal axis passing through its centre with angular velocity ω . A piece of mass m breaks from the disc and flies off vertically upwards. The angular speed of the disc will be

- (a) $\frac{(M-2m)\omega}{(M-m)}$ (b) $\frac{(M+2m)\omega}{(M+m)}$ (c) $\frac{(M-2m)\omega}{(M+m)}$ (d) $\frac{(M+2m)\omega}{(M-m)}$

►► Advance level

83. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved

- (a) Centre of the circle (b) On the circumference of the circle
 (c) Inside the circle (d) Outside the circle

84. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω . Another disc of same dimension but of mass $M/4$ is placed gently on the first disc coaxially. The angular velocity of the system now is

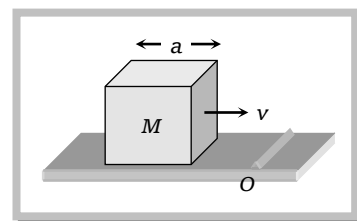
- (a) $2\omega/5$ (b) $2\omega/\sqrt{5}$ (c) $4\omega/5$ (d) $4\omega/\sqrt{5}$

85. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and center of mass with velocity v . It collides elastically and head-on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B respectively. Then

- (a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$ (c) $\omega_A = \omega$ (d) $\omega = \omega_B$

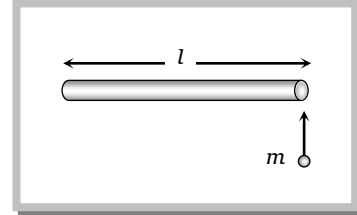
86. A cubical block of side a is moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O . The angular speed of the block after it hits O is

- (a) $3v/4a$
 (b) $3v/2a$
 (c) $\frac{\sqrt{3}v}{\sqrt{2}a}$
 (d) Zero



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87. A stick of length l and mass M lies on a frictionless horizontal surface on which it is free to move in any way. A ball of mass m moving with speed v collides elastically with the stick as shown in the figure. If after the collision ball comes to rest, then what should be the mass of the ball



- (a) $m = 2M$
 (b) $m = M$
 (c) $m = M/2$
 (d) $m = M/4$
88. In a playground there is a merry-go-round of mass 120 kg and radius 4 m . The radius of gyration is 3 m . A child of mass 30 kg runs at a speed of 5 m/sec tangent to the rim of the merry-go-round when it is at rest and then jumps on it. Neglect friction and find the angular velocity of the merry-go-round and child
- (a) 0.2 rad/sec (b) 0.1 rad/sec (c) 0.4 rad/sec (d) 0.8 rad/sec

Problems based on kinetic energy, work and power

► Basic level

89. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass K . If radius of the ball be R , then the fraction of total energy associated with its rotational energy will be
- (a) $\frac{K^2}{R^2}$ (b) $\frac{K^2}{K^2 + R^2}$ (c) $\frac{R^2}{K^2 + R^2}$ (d) $\frac{K^2 + R^2}{R^2}$
90. In a bicycle the radius of rear wheel is twice the radius of front wheel. If r_F and r_r are the radii, v_F and v_r are speeds of top most points of wheel, then
- (a) $v_r = 2 v_F$ (b) $v_F = 2 v_r$ (c) $v_F = v_r$ (d) $v_F > v_r$
91. The total kinetic energy of a body of mass 10 kg and radius 0.5 m moving with a velocity of 2 m/s without slipping is 32.8 joule . The radius of gyration of the body is
- (a) 0.25 m (b) 0.2 m (c) 0.5 m (d) 0.4 m
92. The moment of inertia of a body about a given axis is 2.4 kg-m^2 . To produce a rotational kinetic energy of 750 J , an angular acceleration of 5 rad/s^2 must be applied about that axis for
- (a) 6 sec (b) 5 sec (c) 4 sec (d) 3 sec
93. A solid sphere of mass 500 gm and radius 10 cm rolls without slipping with the velocity 20 cm/s . The total kinetic energy of the sphere will be
- (a) 0.014 J (b) 0.028 J (c) 280 J (d) 140 J
94. The ratio of rotational and translatory kinetic energies of a sphere is [KCET (Med.) 2001; AFMC 2001]
- (a) $\frac{2}{9}$ (b) $\frac{2}{7}$ (c) $\frac{2}{5}$ (d) $\frac{7}{2}$
95. A thin hollow cylinder open at both ends:
- (i) Slides without rotating
 (ii) Rolls without slipping, with the same speed.
- The ratio of kinetic energy in the two cases is [KCET (Engg./Med.) 2000]
- (a) $1 : 1$ (b) $4 : 1$ (c) $1 : 2$ (d) $2 : 1$
96. A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is [MP PMT 1987; BHU 1998]

- (a) $\frac{2}{5}$ (b) $\frac{2}{7}$ (c) $\frac{3}{5}$ (d) $\frac{3}{7}$
97. A body is rolling without slipping on a horizontal plane. If the rotational energy of the body is 40% of the total kinetic energy, then the body might be
 (a) Cylinder (b) Hollow sphere (c) Solid cylinder (d) Ring
98. A body of moment of inertia of $3kg-m^2$ rotating with an angular speed of 2 rad/sec has the same kinetic energy as a mass of 12 kg moving with a speed of
 (a) 1 m/s (b) 2 m/s (c) 4 m/s (d) 8 m/s
99. Ratio of kinetic energy and rotational energy in the motion of a disc is
 (a) $1 : 1$ (b) $2 : 7$ (c) $1 : 2$ (d) $3 : 1$
100. A solid sphere is moving on a horizontal plane. Ratio of its transitional Kinetic energy and rotational energy is [CPMT 1990]
 (a) $1/5$ (b) $5/2$ (c) $3/5$ (d) $5/7$
101. The speed of rolling of a ring of mass M changes from V to $3V$. What is the change in its kinetic energy
 (a) $3MV^2$ (b) $4MV^2$ (c) $6MV^2$ (d) $8MV^2$
102. A disc of radius 1 m and mass 4 kg rolls on a horizontal plane without slipping in such a way that its centre of mass moves with a speed of 10 cm/sec . Its rotational kinetic energy is
 (a) 0.01 erg (b) 0.02 joule (c) 0.03 joule (d) 0.01 joule
103. The ratio of kinetic energies of two spheres rolling with equal centre of mass velocities is $2 : 1$. If their radii are in the ratio $2 : 1$; then the ratio of their masses will be
 (a) $2:1$ (b) $1:8$ (c) $1:7$ (d) $2\sqrt{2} : 1$
104. A symmetrical body of mass M and radius R is rolling without slipping on a horizontal surface with linear speed v . Then its angular speed is
 (a) v/R (b) Continuously increasing
 (c) Dependent on mass M (d) Independent of radius (R)
105. A solid sphere of mass 1 kg rolls on a table with linear speed 1 m/s . Its total kinetic energy is
 (a) 1 J (b) 0.5 J (c) 0.7 J (d) 1.4 J
106. A circular disc has a mass of 1 kg and radius 40 cm . It is rotating about an axis passing through its centre and perpendicular to its plane with a speed of 10 rev/s . The work done in joules in stopping it would be
 (a) 4 (b) 47.5 (c) 79 (d) 158
107. Rotational kinetic energy of a given body about an axis is proportional to
 (a) Time period (b) $(\text{Time period})^2$ (c) $(\text{Time period})^{-1}$ (d) $(\text{time period})^{-2}$
108. If a body completes one revolution in π sec then the moment of inertia would be
 (a) Equal to rotational kinetic energy (b) Double of rotational kinetic energy
 (c) Half of rotational kinetic energy (d) Four times the rotational kinetic energy
109. A tangential force F is applied on a disc of radius R , due to which it deflects through an angle θ from its initial position. The work done by this force would be
 (a) FR (b) $F\theta$ (c) $\frac{FR}{\theta}$ (d) $FR\theta$
110. If the rotational kinetic energy of a body is increased by 300% then the percentage increase in its angular momentum will be
 (a) 600% (b) 150% (c) 100% (d) 1500%
111. A wheel of moment of inertia 10 kg-m^2 is rotating at 10 rotations per minute. The work done in increasing its speed to 5 times its initial value, will be
 (a) 100 J (b) 131.4 J (c) 13.4 J (d) 0.131 J



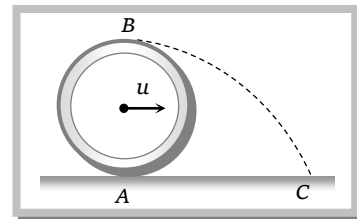
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112. A flywheel has moment of inertia 4 kg-m^2 and has kinetic energy of 200 J . Calculate the number of revolutions it makes before coming to rest if a constant opposing couple of 5 N-m is applied to the flywheel
- (a) 12.8 rev (b) 24 rev (c) 6.4 rev (d) 16 rev
113. An engine develops 100 kW , when rotating at 1800 rpm . Torque required to deliver the power is
- (a) 531 N-m (b) 570 N-m (c) 520 N-m (d) 551 N-m

►► Advance level

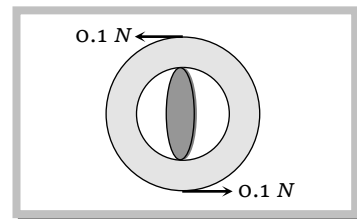
114. A wheel of radius r rolls without slipping with a speed v on a horizontal road. When it is at a point A on the road, a small jump of mud separates from the wheel at its highest point B and drops at point C on the road. The distance AC will be

- (a) $v\sqrt{\frac{r}{g}}$
 (b) $2v\sqrt{\frac{r}{g}}$
 (c) $4v\sqrt{\frac{r}{g}}$
 (d) $\sqrt{\frac{3r}{g}}$



115. A fly wheel of moment of inertia I is rotating at n revolutions per sec. The work needed to double the frequency would be
- (a) $2\pi^2 In^2$ (b) $4\pi^2 In^2$ (c) $6\pi^2 In^2$ (d) $8\pi^2 In^2$
116. If L , M and P are the angular momentum, mass and linear momentum of a particle respectively which of the following represents the kinetic energy of the particle when the particle rotates in a circle of radius R
- (a) $\frac{L^2}{2M}$ (b) $\frac{P^2}{2MR}$ (c) $\frac{L^2}{2MR^2}$ (d) $\frac{MP}{2}$
117. A uniform thin rod of length l is suspended from one of its ends and is rotated at f rotations per second. The rotational kinetic energy of the rod will be
- (a) $\frac{2}{3}\pi^2 f^2 ml^2$ (b) $\frac{4}{3}f^2 ml^2$ (c) $4\pi^2 f^2 ml^2$ (d) Zero
118. A body rotating at 20 rad/sec is acted upon by a constant torque providing it a deceleration of 2 rad/sec^2 . At what time will the body have kinetic energy same as the initial value if the torque continues to act
- (a) 20 secs (b) 40 secs (c) 5 secs (d) 10 secs
119. Part of the tuning arrangement of a radio consists of a wheel which is acted on by two parallel constant forces as shown in the fig. If the wheel rotates just once, the work done will be about (diameter of the wheel = 0.05 m)

- (a) 0.062 J
 (b) 0.031 J
 (c) 0.015 J
 (d) 0.057 J



Problems based on rolling on incline plane

► Basic level

120. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling)
- (a) Solid sphere (b) hollow sphere (c) Ring (d) All same

121. A solid sphere (mass $2M$) and a thin hollow spherical shell (mass M) both of the same size, roll down an inclined plane, then
- [Kerala (Engg.) 2002]
- (a) Solid sphere will reach the bottom first (b) Hollow spherical shell will reach the bottom first
(c) Both will reach at the same time (d) None of these
122. A hollow cylinder and a solid cylinder having the same mass and same diameter are released from rest simultaneously from the top of an inclined plane. Which will reach the bottom first
- (a) The solid cylinder (b) The hollow cylinder
(c) Both will reach the bottom together (d) The greater density
123. The speed of a homogeneous solid sphere after rolling down an inclined plane of vertical height h , from rest without sliding, is
- [CBSE PMT 1992]
- (a) $\sqrt{\frac{10}{7}gh}$ (b) \sqrt{gh} (c) $\sqrt{\frac{6}{5}gh}$ (d) $\sqrt{\frac{4}{3}gh}$
124. A solid cylinder rolls down an inclined plane from a height h . At any moment the ratio of rotational kinetic energy to the total kinetic energy would be
- (a) 1 : 2 (b) 1 : 3 (c) 2 : 3 (d) 1 : 1
125. An inclined plane makes an angle of 30° with the horizontal. A solid sphere rolling down this inclined plane from rest without slipping has a linear acceleration equal to
- (a) $\frac{g}{3}$ (b) $\frac{2g}{3}$ (c) $\frac{5g}{7}$ (d) $\frac{5g}{14}$
126. A solid cylinder of mass M and radius R rolls down an inclined plane without slipping. The speed of its centre of mass when it reaches the bottom is
- (a) $\sqrt{2gh}$ (b) $\sqrt{\frac{4}{3}gh}$ (c) $\sqrt{\frac{3}{4}gh}$ (d) $\sqrt{\frac{g}{h}}$
127. Solid cylinders of radii r_1, r_2 and r_3 roll down an inclined plane from the same place simultaneously. If $r_1 > r_2 > r_3$, which one would reach the bottom first
- (a) Cylinder of radius r_1 (b) Cylinder of radius r_2
(c) Cylinder of radius r_3 (d) All the three cylinders simultaneously
128. A solid cylinder of radius R and mass M , rolls down on inclined plane without slipping and reaches the bottom with a speed v . The speed would be less than v if we use
- (a) A cylinder of same mass but of smaller radius (b) A cylinder of same mass but of larger radius
(c) A cylinder of same radius but of smaller mass (d) A hollow cylinder of same mass and same radius
129. A body starts rolling down an inclined plane of length L and height h . This body reaches the bottom of the plane in time t . The relation between L and t is
- (a) $t \propto L$ (b) $t \propto 1/L$ (c) $t \propto L^2$ (d) $t \propto \frac{1}{L^2}$
130. A hollow cylinder is rolling on an inclined plane, inclined at an angle of 30° to the horizontal. Its speed after travelling a distance of 10 m will be
- (a) 49 m/sec (b) 0.7 m/sec (c) 7 m/sec (d) Zero
131. A solid sphere, a solid cylinder, a disc and a ring are rolling down an inclined plane. Which of these bodies will reach the bottom simultaneously
- (a) Solid sphere and solid cylinder (b) Solid cylinder and disc
(c) Disc and ring (d) Solid sphere and ring
132. A ball of radius 11 cm and mass 8 kg rolls from rest down a ramp of length 2 m . The ramp is inclined at 35° to the horizontal. When the ball reaches the bottom, its velocity is ($\sin 35^\circ = 0.57$)
- (a) 2 m/s (b) 5 m/s (c) 4 m/s (d) 6 m/s



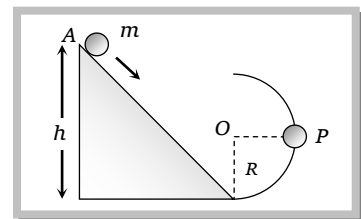
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133. From an inclined plane a sphere, a disc, a ring and a shell are rolled without slipping. The order of their reaching at the base will be
 (a) Ring, shell, disc, sphere (b) Shell, sphere, disc, ring (c) Sphere, disc, shell, ring (d)
134. A solid cylinder 30 cm in diameter at the top of an inclined plane 2.0 m high is released and rolls down the incline without loss of energy due to friction. Its linear speed at the bottom is
 (a) 5.29 m/sec (b) 4.1×10^3 m/sec (c) 51 m/sec (d) 51 cm/sec
135. A cylinder of mass M and radius R rolls on an inclined plane. The gain in kinetic energy is
 (a) $\frac{1}{2}Mv^2$ (b) $\frac{1}{2}I\omega^2$ (c) $\frac{3}{4}Mv^2$ (d) $\frac{3}{4}I\omega^2$
136. A disc of radius R is rolling down an inclined plane whose angle of inclination is θ , Its acceleration would be
 (a) $\frac{5}{7}g \sin \theta$ (b) $\frac{2}{3}g \sin \theta$ (c) $\frac{1}{2}g \sin \theta$ (d) $\frac{3}{5}g \sin \theta$
137. A solid cylinder (i) rolls down (ii) slides down an inclined plane. The ratio of the accelerations in these conditions is
 (a) 3 : 2 (b) 2 : 3 (c) $\sqrt{3} : \sqrt{2}$ (d) $\sqrt{2} : \sqrt{3}$
138. The acceleration of a body rolling down on an inclined plane does not depend upon
 (a) Angle of inclination of the plane (b) Length of plane
 (c) Acceleration due to gravity of earth (d) Radius of gyration of body
139. A ring, a solid sphere, a disc and a solid cylinder of same radii roll down an inclined plane, which would reach the bottom in the last
 (a) Ring (b) Disc (c) Solid sphere (d) Solid cylinder
140. A ring is rolling on an inclined plane. The ratio of the linear and rotational kinetic energies will be
 (a) 2 : 1 (b) 1 : 2 (c) 1 : 1 (d) 4 : 1
141. The moment of inertia of a solid cylinder about its axis is I . It is allowed to roll down an incline without slipping. If its angular velocity at the bottom be ω , then K.E. of the cylinder will be
 (a) $I\omega^2$ (b) $\frac{3}{2}I\omega^2$ (c) $2I\omega^2$ (d) $\frac{1}{2}I\omega^2$

►► Advance level

142. A solid ball of mass m and radius r rolls without slipping along the track shown in the fig. The radius of the circular part of the track is R . The ball starts rolling down the track from rest from a height of $8R$ from the ground level. When the ball reaches the point P then its velocity will be

- (a) \sqrt{gR}
 (b) $\sqrt{5gR}$
 (c) $\sqrt{10gR}$
 (d) $\sqrt{3gR}$

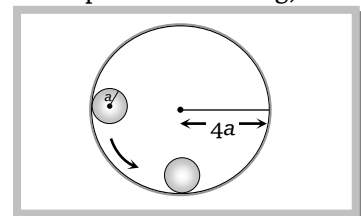


143. A ring takes time t_1 in slipping down an inclined plane of length L and takes time t_2 in rolling down the same plane. The ratio $\frac{t_1}{t_2}$ is

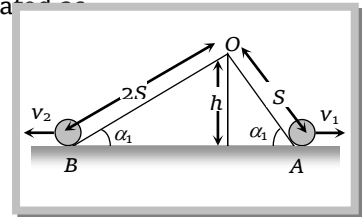
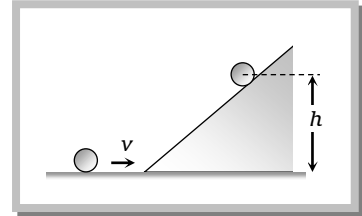
- (a) $\sqrt{2} : 1$ (b) $1 : \sqrt{2}$ (c) 1 : 2 (d) 2 : 1

144. A ring of radius $4a$ is rigidly fixed in vertical position on a table. A small disc of mass m and radius a is released as shown in the fig. When the disc rolls down, without slipping, to the lowest point of the ring, then its speed will be

- (a) \sqrt{ga}
 (b) $\sqrt{2ga}$



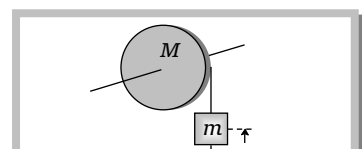
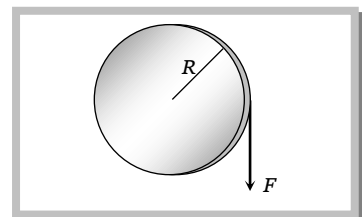
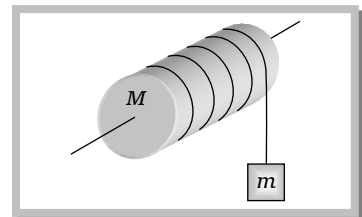
- (c) $\sqrt{3ga}$
 (d) $\sqrt{4ga}$
145. A disc of mass M and radius R rolls in a horizontal surface and then rolls up an inclined plane as shown in the fig. If the velocity of the disc is v , the height to which the disc will rise will be
- (a) $\frac{3v^2}{2g}$
 (b) $\frac{3v^2}{4g}$
 (c) $\frac{v^2}{4g}$
 (d) $\frac{v^2}{2g}$
146. Two uniform similar discs roll down two inclined planes of length S and $2S$ respectively as shown is the fig. The velocities of two discs at the points A and B of the inclined planes are related as
- (a) $v_1 = v_2$
 (b) $v_1 = 2v_2$
 (c) $v_1 = v_2 \frac{v_2}{4}$
 (d) $v_1 = \frac{3}{4}v_2$



Problems based on motion of connected mass

► Basic level

147. A mass M is supported by a mass less string wound a uniform cylinder of mass M and radius R . On releasing the mass from rest, it will fall with acceleration
- (a) g
 (b) $\frac{g}{2}$
 (c) $\frac{g}{3}$
 (d) $\frac{2g}{3}$
148. A uniform disc of radius R and mass M can rotate on a smooth axis passing through its centre and perpendicular to its plane. A force F is applied on its rim. See fig. What is the tangential acceleration
- (a) $\frac{2F}{M}$
 (b) $\frac{F}{M}$
 (c) $\frac{F}{2M}$
 (d) $\frac{F}{4M}$
149. A massless string is wrapped round a disc of mass M and radius R . Another end is tied to a mass m which is initially at height h from ground level as shown in the fig. If the mass is released then its velocity while touching the ground level will be
- (a) $\sqrt{2gh}$

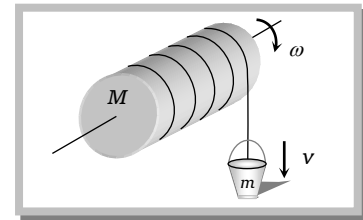


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- (b) $\sqrt{2gh} \frac{M}{m}$
 (c) $\sqrt{2ghm / M}$
 (d) $\sqrt{4mgh / 2m + M}$

150. A cylinder of mass M and radius r is mounted on a frictionless axle over a well. A rope of negligible mass is wrapped around the cylinder and a bucket of mass m is suspended from the rope. The linear acceleration of the bucket will be

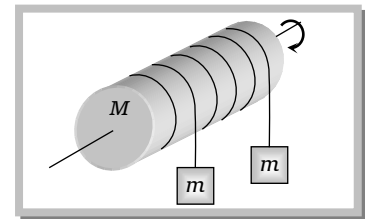
- (a) $\frac{Mg}{M + 2m}$
 (b) $\frac{2Mg}{m + 2M}$
 (c) $\frac{Mg}{2M + m}$
 (d) $\frac{2mg}{M + 2m}$



►► Advance level

151. A uniform solid cylinder of mass M and radius R rotates about a frictionless horizontal axle. Two similar masses suspended with the help two ropes wrapped around the cylinder. If the system is released from rest then the acceleration of each mass will be

- (a) $\frac{4mg}{M + 2m}$
 (b) $\frac{4mg}{M + 4m}$
 (c) $\frac{2mg}{M + m}$
 (d) $\frac{2mg}{M + 2m}$



152. In the above problem the angular velocity of the cylinder, after the masses fall down through distance h , will be

- (a) $\frac{1}{R} \sqrt{8mgh / (M + 4m)}$ (b) $\frac{1}{R} \sqrt{8mgh / (M + m)}$ (c) $\frac{1}{R} \sqrt{mgh / (M + m)}$ (d) $\frac{1}{R} \sqrt{8mgh / (M + 2m)}$

Problems based on compound pendulum

► Basic level

153. A metre scale is suspended vertically from a horizontal axis passing through one end of it. Its time period would be

- (a) 1.64 sec (b) 2 sec (c) 2.5 sec (d) 3.2 sec

154. A disc of radius R is made to oscillate about a horizontal axis passing through its periphery. Its time period would be

- (a) $2\pi \sqrt{\frac{3R}{2g}}$ (b) $2\pi \sqrt{\frac{2R}{3g}}$ (c) $2\pi \sqrt{\frac{R}{g}}$ (d) $2\pi \sqrt{\frac{2R}{g}}$

►► Advance level

155. A solid cube of side l is made to oscillate about a horizontal axis passing through one of its edges. Its time period will be

(a) $2\pi\sqrt{\frac{2\sqrt{2}}{3}\frac{l}{g}}$

(b) $2\pi\sqrt{\frac{2}{3}\frac{l}{g}}$

(c) $2\pi\sqrt{\frac{\sqrt{3}}{2}\frac{l}{g}}$

(d) $2\pi\sqrt{\frac{2}{\sqrt{3}}\frac{l}{g}}$

156. The string of a simple pendulum is replaced by a uniform rod of length L and mass M . If the mass of the bob of the pendulum is m , then for small oscillations its time period would be (assume radius of bob $r \ll L$)

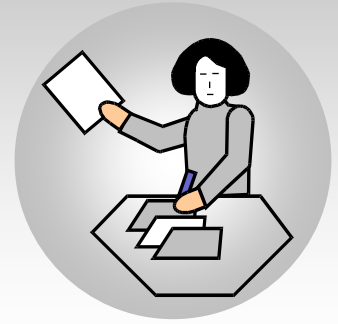
(a) $2\pi\sqrt{\frac{2(M+3m)L}{3(M+2m)g}}$

(b) $2\pi\sqrt{\frac{(M+2m)L}{3(M+3m)g}}$

(c) $2\pi\sqrt{\left(\frac{2M}{3m}\right)\frac{L}{g}}$

(d) $2\pi\sqrt{\left(\frac{M+m}{M+3m}\right)\frac{L}{g}}$





Answer Sheet (Practice problems)

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
b	c	c	b	c	a	a	a	c	c
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
b	a	c	c	a	a	a	b	c	d
21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
a	d	c	b	b	c	d	c	a	b
31.	32.	33.	34.	35.	36.	37.	38.	39.	40.
d	c	c	c	c	d	b	c	b	c
41.	42.	43.	44.	45.	46.	47.	48.	49.	50.
b	a	b	a	a	d	a	b	a	d
51.	52.	53.	54.	55.	56.	57.	58.	59.	60.
b	d	d	d	b	a	c	d	b	c
61.	62.	63.	64.	65.	66.	67.	68.	69.	70.
d	a	b	d	a	b	c	d	b	a
71.	72.	73.	74.	75.	76.	77.	78.	79.	80.
c	b	c	d	b	c	c	b	b	b
81.	82.	83.	84.	85.	86.	87.	88.	89.	90.
c	a	a	c	c	a	d	c	b	c
91.	92.	93.	94.	95.	96.	97.	98.	99.	100.
d	b	a	c	c	b	b	a	d	b
101.	102.	103.	104.	105.	106.	107.	108.	109.	110.
d	d	a	a	c	d	d	c	d	c
111.	112.	113.	114.	115.	116.	117.	118.	119.	120.
b	c	a	c	c	c	a	a	b	d
121.	122.	123.	124.	125.	126.	127.	128.	129.	130.
a	a	a	b	d	b	d	d	a	c
131.	132.	133.	134.	135.	136.	137.	138.	139.	140.
b	c	c	a	c	b	b	b	a	c
141.	142.	143.	144.	145.	146.	147.	148.	149.	150.
b	c	b	d	b	a	d	a	d	d
151.	152.	153.	154.	155.	156.				
b	a	a	a	a	a				

